Welcome back to the world of calculations.
In Tutorial 10 you will be shown:

1. What is meant by \( K_{sp} \).

2. How to write a "\( K_{sp} \) expression" from a net ionic equation.

3. How to calculate the solubility of an ionic substance in moles/L or in grams/L, given the value of \( K_{sp} \).

4. How to calculate the value of \( K_{sp} \), given the solubility of an ionic substance.

What is \( K_{sp} \)?

\( K_{sp} \) is really nothing new. It's simply the equilibrium constant (\( K_{eq} \)) for an ionic substance dissolving in water.

For example, for the substance, \( \text{CaCO}_3(\text{s}) \), when it dissolves in water, forms the ions \( \text{Ca}^{2+} \) and \( \text{CO}_3^{2-} \).

The equilibrium net-ionic equation for this is: \( \text{CaCO}_3(\text{s}) \rightleftharpoons \text{Ca}^{2+}(\text{aq}) + \text{CO}_3^{2-}(\text{aq}) \).

The equilibrium constant (\( K_{eq} \)) expression for this reaction is:

\[
K_{eq} = [\text{Ca}^{2+}] [\text{CO}_3^{2-}] \quad (\text{Remember that the } \{\text{CaCO}_3\} \text{ is not included in the expression because it is a solid.})
\]

Remember that the solubility was defined as the equilibrium concentration of a substance in water.

But when ionic compounds dissolve in water, you always get at least 2 ions formed. (In this case the \( \text{Ca}^{2+} \) and the \( \text{CO}_3^{2-} \)).

Multiplying the concentrations of the two ions (multiplication gives a "product"), as you can see, gives the \( K_{eq} \) expression:

\[
K_{eq} = [\text{Ca}^{2+}] [\text{CO}_3^{2-}] 
\]

\( K_{eq} \) can be thought of as the "product of the solubility’s of the two ions". So for ionic compounds dissolving in water, the \( K_{eq} \) is given a special name:

It is called "solubility product constant", or \( K_{sp} \).
Writing the Ksp expression from the Net-Ionic Equation.

There's really nothing new to this. Just remember to leave out the solid. (Include the aqueous ions only) and to change coefficients in the balanced equation to exponents in the Ksp expression.

Here are a couple of examples:

The net-ionic equation for lead (II) chloride dissolving is:

\[
PbCl_2(s) \rightleftharpoons Pb^{2+}(aq) + 2Cl^-(aq)
\]

The Ksp expression for this is:

\[
K_{sp} = [Pb^{2+}] [Cl^-]^2
\]

(Notice that the "2" in front of the Cl\(^-\) in the equation becomes an exponent in the Ksp expression. It does NOT go in front of the Cl\(^-\) in the Ksp expression!)

The net-ionic equation for silver sulphate dissolving is:

\[
Ag_2SO_4(s) \rightleftharpoons 2Ag^+(aq) + SO_4^{2-}(aq)
\]

The Ksp expression would be:

\[
K_{sp} = [Ag^+]^2 [SO_4^{2-}]
\]

I don't think you should have any more trouble with these as long as you follow the simple rules:

1. leave out the solid. (Include the aqueous ions only)

2. change coefficients in the balanced equation to exponents in the Ksp expression.

Calculating Solubility given Ksp.

The first type of calculation we will look at is how to calculate the solubility of a substance in moles per litre (M), given the value of Ksp.

NOTE: We only consider the Ksp and the solubility of substances which have "Low Solubility" on the Solubility Table. These are also called "Slightly Soluble Salts"

For these calculations, we can define molar solubility as the moles of the substance which will dissolve in one litre of solution to form a saturated solution.

On the next page, you will be shown how to calculate the molar solubility of AgCl from the Ksp.
First we obtain a sheet entitled “Solubility Product Constants at 25 °C” This has Ksp’s for many of the “Low Solubility Compounds” listed. Always have this table with you on a test!

So, we find that the Ksp for AgCl = 1.8 x 10^{-10}

The net-ionic equation for the substance, AgCl dissolving in water is:

\[ \text{AgCl(s)} \rightleftharpoons \text{Ag}^{+}(aq) + \text{Cl}^{-}(aq) \]

The number of moles of AgCl which dissolve in one litre can be defined as "s". (This is the molar solubility.)

Notice, that this is written as "-s" above AgCl in the equation. This is because when you dissolve AgCl in water, the amount of solid will go down by "s" because that much dissolves. The “-s” is the same thing as “[C]” or “change in concentration” in an ICE table.

Now by mole ratios, you can see, that in this case [Ag+] will increase by "s" as the AgCl dissolves. Also, the [Cl-] will also increase by "s". (The coefficients of Ag+ and Cl- are both "1".)

\[ \frac{1}{1} \]

\[ \text{AgCl(s)} \rightleftharpoons \text{Ag}^{+}(aq) + \text{Cl}^{-}(aq) \]

\[ \frac{1}{1} \]

\[ \text{AgCl(s)} \rightleftharpoons \text{Ag}^{+}(aq) + \text{Cl}^{-}(aq) \]

so we have:

\[ \text{AgCl(s)} \rightleftharpoons \text{Ag}^{+}(aq) + \text{Cl}^{-}(aq) \]

where "s" is the moles/L of AgCl which dissolve.

The Ksp expression for this process is:

\[ \text{Ksp} = [\text{Ag}^{+}] [\text{Cl}^{-}] \]

What we do now is "plug in” the values of [Ag+] and [Cl-] in terms of "s". These are the expressions written on top of the net-ionic equation:

\[ [\text{Ag}^{+}] = s \text{ and } [\text{Cl}^{-}] = s \text{ (we don't need to write the "+" signs)} \]
Since: \( K_{sp} = [Ag^+] [Cl^-] \)

\[ K_{sp} = (s) (s) \]

or \( K_{sp} = s^2 \)

Now we can solve for \( s \). Remember we found from the \( K_{sp} \) table that the value for \( K_{sp} = 1.8 \times 10^{-10} \)

So \( s^2 = 1.8 \times 10^{-10} \) or \( s = \sqrt{1.8 \times 10^{-10}} = 1.34164 \times 10^{-5} \) M

Since "s" was defined as the molar solubility of AgCl:

\[
\begin{align*}
-\text{s} & \quad \Leftrightarrow \quad +\text{s} \\
\text{AgCl}^{(s)} & \quad \text{Ag}^{+}^{(aq)} + \text{Cl}^{-}^{(aq)}
\end{align*}
\]

where "s" is the moles/L of AgCl which dissolve.

We can say, the molar solubility of AgCl is \( 1.34164 \times 10^{-5} \) moles/L or \( 1.3 \times 10^{-5} \) M (2 sig. digs.) (The \( K_{sp} \) given on the table has 2 SD’s)

NOTE: It looks awful tempting at this point to just say that the solubility is the square root of the \( K_{sp} \). It is in this case, but not in all cases.

Make sure when you do these problems that you always write:

1. The net-ionic equilibrium equation (with the "s"s on top according to the coefficients),

2. The \( K_{sp} \) expression, and

3. The concentrations plugged into the \( K_{sp} \) expression to solve for "s".

DON’T TAKE SHORT CUTS! They could lead to trouble in the next type of problem!

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The next problem, you should try on your own, keeping the last three points in mind. Don’t worry too much if you get it wrong the first time. Just make sure you understand the solution given on page 1 of Tutorial 10 - Solutions.

1. Calculate the solubility of SrF$_2$ in moles/Litre in water.

Answer ______________________

Check your answer on page 1 of Tutorial 10 – Solutions
Now, let's say you're given the Ksp table and asked to find the solubility of a substance in grams/L.

What you would do is determine the solubility in moles/L (or M) first, then convert to grams/L using the following:

\[
\text{moles} \times \frac{MM}{1 \text{ mole}} = \frac{\text{grams}}{\text{Litre}}
\]

Where "MM" stands for the Molar Mass.

Try this example:

2. Calculate the solubility of Ag\(_2\)CO\(_3\) in grams/Litre.

Answer ______________________________

Check your answer on page 2 of Tutorial 10 - Solutions

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**Calculating Ksp, Given Solubility**

The first example we will do is to calculate the Ksp of a substance given it's molar solubility.

In this type of problem we don't use "s"s. The molar solubility is known, so we find the concentration of each ion using mole ratios (record them on top of the equation). Next we write out the expression for Ksp, then "plug in" the concentrations to obtain the value for Ksp.

Let's do an example:

The solubility of Ag$_2$CrO$_4$ in water is $1.31 \times 10^{-4}$ moles/L. Calculate the value of Ksp.

The first thing we do is write out the net-ionic equation for a saturated solution of Ag$_2$CrO$_4$:

$$\text{Ag}_2\text{CrO}_4(s) \rightleftharpoons 2\text{Ag}^+(aq) + \text{CrO}_4^{2-}(aq)$$

The solubility given is $1.31 \times 10^{-4}$ moles/L, so we write that right on top of the Ag$_2$CrO$_4(s)$:

(I usually write it as a negative (-) because it is going down (the solid is dissolving))

$$-1.31 \times 10^{-4} \text{ moles/L} \quad \text{Ag}_2\text{CrO}_4(s) \rightleftharpoons 2\text{Ag}^+(aq) + \text{CrO}_4^{2-}(aq)$$

Using mole ratios, the $[\text{Ag}^+]$ will go up by $(2 \times 1.31 \times 10^{-4}$ moles/L) = $2.62 \times 10^{-4}$ moles/L.

$$x \frac{2}{1}$$

$$-1.31 \times 10^{-4} \text{ moles/L} \quad \text{Ag}_2\text{CrO}_4(s) \rightleftharpoons 2\text{Ag}^+(aq) + \text{CrO}_4^{2-}(aq) \quad \rightarrow \quad + \ 2.62 \times 10^{-4} \text{ moles/L}$$

$[\text{CrO}_4^{2-}]$ will go up by $1.31 \times 10^{-4}$ moles/L:

$$x \frac{1}{1}$$

$$-1.31 \times 10^{-4} \text{ moles/L} \quad \text{Ag}_2\text{CrO}_4(s) \rightleftharpoons 2\text{Ag}^+(aq) + \text{CrO}_4^{2-}(aq) \quad \rightarrow \quad + \ 1.31 \times 10^{-4} \text{ M.}$$

So once the solution is saturated (reaches equilibrium):

$[\text{Ag}^+] = 2.62 \times 10^{-4} \text{ moles/L}$

$[\text{CrO}_4^{2-}] = 1.31 \times 10^{-4} \text{ moles/L}$
The Ksp expression is:

\[ K_{sp} = [Ag^+]^2 [CrO_4^{2-}] \]

\[ [Ag^+] = 2.62 \times 10^{-4} \text{ moles/L} \]
\[ [CrO_4^{2-}] = 1.31 \times 10^{-4} \text{ moles/L} \]

Plugging in the concentrations directly we get:

\[ K_{sp} = (2.62 \times 10^{-4})^2 (1.31 \times 10^{-4}) \]

\[ K_{sp} = 8.99 \times 10^{-12} \]

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Now, here’s a question for you to try:

3. At a certain temperature, the solubility of SrCO\textsubscript{3} is 7.5 \times 10^{-5} \text{ M}. Calculate the \textit{Ksp} for SrCO\textsubscript{3}.

Answer __________________

Check your answer on Page 4 of Tutorial 10 - Solutions

The next type of problem involves calculating the value of Ksp given the solubility in grams per Litre.

For this, you simply change grams/L to moles/L using the following:

\[
\frac{\text{grams}}{\text{Litre}} \times \frac{1 \text{ mole}}{M\text{M grams}} = \frac{\text{moles}}{\text{Litre}}
\]

where "MM" stands for the Molar Mass.

Then write out the Net-Ionic equation for the equilibrium, and the Ksp expression.
Determine the concentrations of the ions and plug them into the Ksp expression to solve for Ksp.

Here's an example:

6.60 grams of MnF$_2$ will dissolve in one Litre of solution at 25°C. Calculate the value of Ksp for MnF$_2$ at 25°C.

**Step 1- Change solubility in grams/L to moles/L using Molar Mass:**

The molar mass of MnF$_2$ = 54.9 + 2(19.0) = 92.9 grams/mole

\[
\begin{align*}
6.60 \text{ grams} & \quad x \quad 1 \text{ mole} \quad = \quad 0.071044 \text{ moles} \\
\text{Litre} & \quad 92.9 \text{ grams} & \quad \text{Litre}
\end{align*}
\]

So, from step 1, the molar solubility of MnF$_2$ is 0.071044 moles/Litre.

**Step 2- Write out the Net-Ionic Equation:**

MnF$_2$(s) $\rightleftharpoons$ Mn$^{2+}$(aq) + 2F$^-$ (aq)

**Step 3 - Write the molar solubility above the MnF$_2$(s) and determine the [Mn$^{2+}$] and [F$^-$] using mole ratios:**

\[
\begin{align*}
-0.071044 \text{ moles/L} & \quad \rightleftharpoons \quad +0.071044 \text{ moles/L} \\
1 \text{ MnF}_2(s) & \quad \rightleftharpoons \quad 1 \text{ Mn}^{2+}(aq) + \quad 2\text{F}^-(aq)
\end{align*}
\]

\[
\begin{align*}
-0.071044 \text{ moles/L} & \quad \rightleftharpoons \quad +0.14209 \text{ moles/L} \\
1 \text{ MnF}_2(s) & \quad \rightleftharpoons \quad 1 \text{ Mn}^{2+}(aq) + \quad 2\text{F}^-(aq)
\end{align*}
\]

It is important that you understand here that [F$^-$] is 0.14209, not the \([2\text{F}^-]\)!

\((\text{[F}^-\text{]} \text{ happens to be 2 times the solubility of MnF}_2 \text{ just because of the mole ratio in the balanced equation.})\)

So [Mn$^{2+}$] = 0.071044 M and [F$^-$] = 0.14209 M

**Step 4 - Write out the Ksp expression:**

\[
\text{Ksp} = [\text{Mn}^{2+}] \quad [\text{F}^-]^2 \quad (\text{not} \quad [2\text{F}^-])
\]
Step 5 - "Plug-in" the values for [Mn^{2+}] and [F^-] and solve for the value of Ksp.

\[
[Mn^{2+}] = 0.071044 \text{ M} \quad \text{and} \quad [F^-] = 0.14209 \text{ M}
\]

\[
Ksp = [Mn^{2+}] \times [F^-]^2 = (0.071044) \times (0.14209)^2
\]

\[
Ksp = 1.43 \times 10^{-3}
\]

In summary, we have covered the following:

1. What is meant by \(Ksp\)?

2. How to write a "\(Ksp\) expression" from a net ionic equation.

3. How to calculate the solubility of an ionic substance in moles/L or in grams/L, given Ksp.

4. How to calculate the value of Ksp, given the solubility of an ionic substance.

**Self-Test on Tutorial 10**

Check your answers starting on page 5 of Tutorial 10 - Solutions

1. The Ksp is just a _________ for an ionic substance dissolving in water.

2. Give the Net-Ionic Equation and the Ksp expression for each of the following dissolving in water. (The first one is done as an example.)

<table>
<thead>
<tr>
<th>Substance</th>
<th>Net-Ionic Equation</th>
<th>Ksp Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ag_2SO_4(s)</td>
<td>(2Ag^+(aq) + SO_4^{2-}(aq))</td>
<td>(Ksp = [Ag^+]^2[SO_4^{2-}])</td>
</tr>
<tr>
<td>CaCO_3(s)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ca_3(PO_4)_2(s)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AgClO_3(s)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The lowest # of SD’s in the data was 3 SD’s. As we were working out the problem, we kept more SD’s than this. Then, in the very last step, we rounded the answer to 3 SD’s.
Chemistry 12

Unit 3 - Solubility of Ionic Substances

3. a) Calculate the molar solubility (solubility in moles/Litre) of Fe(OH)$_2$ in water.

Answer ______________________

b) What is the [OH$^-$] in a saturated solution of Fe(OH)$_2$?

Answer ______________________

4. Calculate the solubility of BaCO$_3$ in grams per Litre.

Answer ______________________
5. The solubility of PbI$_2$ at a certain temperature is 0.70 grams per Litre.

   a) Calculate the solubility in moles/Litre

   Answer _____________________

   b) Calculate the value of Ksp for PbI$_2$ at this temperature

   Answer _____________________

6. It is found that 0.043 grams of MgCO$_3$ is all that can dissolve in 100.0 mL of solution at a certain temperature. From this information, calculate the Ksp for MgCO$_3$ at this temperature.

   Answer _____________________
7. Two separate experiments were done with combinations of Cu$^{2+}$ and IO$_3^-$ ions. Use the information given to fill in the missing value.

The Net-Ionic Equation for equilibrium is: $\text{Cu(IO}_3\text{)}_{2(s)} \rightleftharpoons \text{Cu}^{2+}_{(aq)} + 2\text{IO}_3^-_{(aq)}$

<table>
<thead>
<tr>
<th>Experiment #</th>
<th>[Cu$^{2+}$]</th>
<th>[IO$_3^-$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00327 M</td>
<td>0.00654 M</td>
</tr>
<tr>
<td>2</td>
<td>0.00240 M</td>
<td>?</td>
</tr>
</tbody>
</table>

Check your answers starting on page 5 of Tutorial 10 - Solutions

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This is the End of Tutorial 10