

Chemistry 12
Tutorial 10
K_{sp} Calculations

Welcome back to the world of calculations.

In Tutorial 10 you will be shown:

1. What is meant by **K_{sp}** .
2. How to write a "**K_{sp} expression**" from a *net ionic equation*.
3. How to calculate the **solubility** of an ionic substance in moles/L or in grams/L, given the value of K_{sp} .
4. How to calculate the **value** of K_{sp} , given the **solubility** of an ionic substance.

What is K_{sp} ?

K_{sp} is really nothing new. It's simply the equilibrium constant (K_{eq}) for **an ionic substance dissolving in water**.

For example, for the substance, CaCO_{3(s)}, when it dissolves in water, forms the ions Ca²⁺ and CO₃²⁻.

The equilibrium **net-ionic equation** for this is: CaCO_{3(s)} ⇌ Ca²⁺_(aq) + CO₃²⁻_(aq).

The equilibrium constant (K_{eq}) expression for this reaction is:

$$K_{eq} = [Ca^{2+}] [CO_3^{2-}] \quad (\text{Remember that the } [CaCO_3] \text{ is not included in the expression because it is a solid.)$$

Remember that the **solubility** was defined as *the equilibrium concentration of a substance in water*.

But when ionic compounds dissolve in water, you always get *at least 2 ions* formed. (In this case the Ca²⁺ and the CO₃²⁻.)

Multiplying the concentrations of the two ions (multiplication gives a "**product**"), as you can see, gives the K_{eq} expression:

$$K_{eq} = [Ca^{2+}] [CO_3^{2-}]$$

K_{eq} can be thought of as the "product of the solubility's of the two ions". So for ionic compounds dissolving in water, the K_{eq} is given a special name:

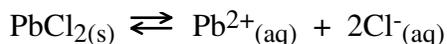
It is called "**solubility product constant**", or **K_{sp}** .

Writing the K_{sp} expression from the Net-Ionic Equation.

There's really nothing new to this. Just remember to leave out the solid. (Include the aqueous ions only) and to change coefficients in the balanced equation to exponents in the K_{sp} expression.

Here are a couple of examples:

The net-ionic equation for lead (II) chloride dissolving is:



The K_{sp} expression for this is:

$K_{sp} = [\text{Pb}^{2+}] [\text{Cl}^{-}]^2$ (Notice that the "2" in front of the Cl⁻ in the equation becomes an exponent in the K_{sp} expression. It does NOT go in front of the Cl⁻ in the K_{sp} expression!)

The net-ionic equation for silver sulphate dissolving is:



The K_{sp} expression would be:

$$K_{sp} = [\text{Ag}^{+}]^2 [\text{SO}_4^{2-}]$$

I don't think you should have any more trouble with these as long as you follow the simple rules:

1. leave out the solid. (Include the aqueous ions only)
2. change coefficients in the balanced equation to exponents in the K_{sp} expression.

Calculating Solubility given K_{sp} .

The first type of calculation we will look at is how to calculate the solubility of a substance in moles per litre (M), given the value of K_{sp} .

NOTE: We only consider the K_{sp} and the solubility of substances which have "**Low Solubility**" on the Solubility Table. These are also called "**Slightly Soluble Salts**"

For these calculations, we can define **molar solubility** as the moles of the substance which will dissolve in one litre of solution to form a saturated solution.

On the next page, you will be shown how to calculate the **molar solubility** of AgCl from the K_{sp}.

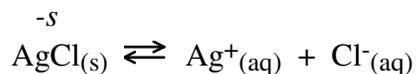
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Unit 3 - Solubility of Ionic Substances

First we obtain a sheet entitled "Solubility Product Constants at 25 °C" This has K_{sp}'s for many of the "Low Solubility Compounds" listed. Always have this table with you on a test!

So, we find that the K_{sp} for AgCl = 1.8 x 10⁻¹⁰

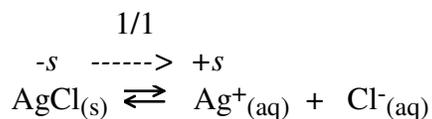
The net-ionic equation for the substance, AgCl dissolving in water is:



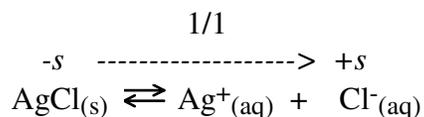
The number of *moles* of AgCl which dissolve in *one litre* can be defined as "*s*". (*This is the molar solubility.*)

Notice, that this is written as "-s" above AgCl in the equation. This is because when you dissolve AgCl in water, the amount of solid will go *down* by "s" because *that much dissolves*. The "-s" is the same thing as "[C]" or "change in concentration" in an ICE table.

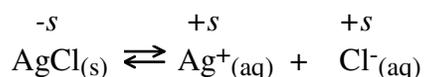
Now by mole ratios, you can see, that in this case [Ag⁺] will *increase* by "s" as the AgCl dissolves. Also, the [Cl⁻] will also *increase* by "s". (*The coefficients of Ag⁺ and Cl⁻ are both "1".*)



and



so we have:



where "s" is the moles/L of AgCl which dissolve.

The K_{sp} expression for this process is:

$$K_{sp} = [\text{Ag}^+] [\text{Cl}^-]$$

What we do now is "plug in" the values of [Ag⁺] and [Cl⁻] in terms of "s". These are the expressions written on top of the net-ionic equation:

$$[\text{Ag}^+] = s \text{ and}$$

$$[\text{Cl}^-] = s \text{ (we don't need to write the "+" signs)}$$

Since: $K_{sp} = [Ag^+][Cl^-]$

$$K_{sp} = (s)(s)$$

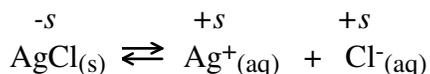
or $K_{sp} = s^2$

Now we can solve for s. Remember we found from the K_{sp} table that the value for

$$K_{sp} = 1.8 \times 10^{-10}$$

$$\text{So } s^2 = 1.8 \times 10^{-10} \text{ or } s = \sqrt{1.8 \times 10^{-10}} = 1.34164 \times 10^{-5} \text{ M}$$

Since "s" was defined as the **molar solubility** of AgCl:



where "s" is the moles/L of AgCl which dissolve.

We can say, **the molar solubility of AgCl** is 1.34164×10^{-5} moles/L or **1.3×10^{-5} M** (2 sig. digs.) (The K_{sp} given on the table has 2 SD's)

NOTE: It looks awful tempting at this point to just say that the solubility is the square root of the K_{sp} . It is in this case, but not in all cases.

Make sure when you do these problems that you always write:

1. The net-ionic equilibrium equation (with the "s"s on top according to the coefficients),
2. The K_{sp} expression, and
3. The concentrations plugged into the K_{sp} expression to solve for "s".

DON'T TAKE SHORT CUTS! They could lead to **trouble** in the next type of problem!

The next problem, you should try on your own, keeping the last three points in mind. Don't worry too much if you get it wrong the first time. Just make sure you understand the solution given on page 1 of Tutorial 10 - Solutions.

1. Calculate the solubility of SrF_2 in moles/Litre in water.

Answer _____

Check your answer on page 1 of Tutorial 10 – Solutions

Now, let's say you're given the K_{sp} table and asked to find the solubility of a substance in grams/L.

What you would do is determine the solubility in moles/L (or M) first, then convert to grams/L using the following:

$$\frac{\text{moles}}{\text{Litre}} \quad \times \quad \frac{MM \text{ grams}}{1 \text{ mole}} = \frac{\text{grams}}{\text{Litre}}$$

Where "MM" stands for the Molar Mass.

Try this example:

2. Calculate the solubility of Ag₂CO₃ in grams/Litre.

Answer _____

Check your answer on page 2 of Tutorial 10 - Solutions

Calculating K_{sp} , Given Solubility

The first example we will do is to calculate the K_{sp} of a substance given its molar solubility.

In this type of problem we don't use "s"s. The molar solubility is known, so we find the concentration of each ion using mole ratios (record them on top of the equation). Next we write out the expression for K_{sp} , then "plug in" the concentrations to obtain the value for K_{sp}.

Let's do an example:

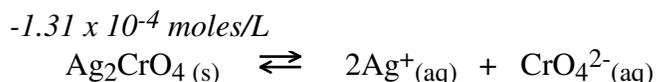
The solubility of Ag₂CrO₄ in water is 1.31 x 10⁻⁴ moles/L. Calculate the value of K_{sp} .

The first thing we do is write out the net-ionic equation for a saturated solution of Ag₂CrO₄:

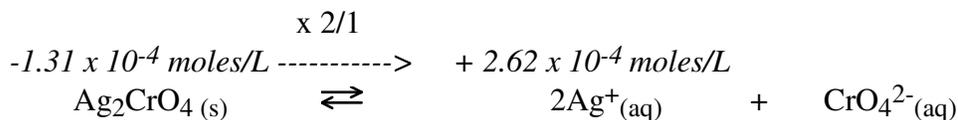


The solubility given is 1.31 x 10⁻⁴ moles/L, so we write that right on top of the Ag₂CrO₄(s)

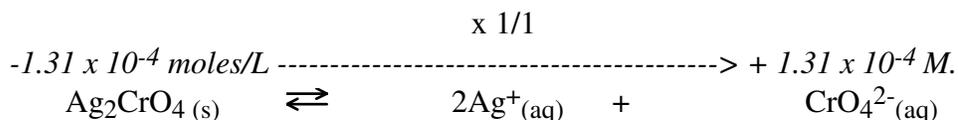
(I usually write it as a negative (-) because it is going down (the solid is dissolving))



Using mole ratios, the [Ag⁺] will go up by (2 x 1.31 x 10⁻⁴ moles/L) = 2.62 x 10⁻⁴ moles/L.



[CrO₄²⁻] will go up by 1.31 x 10⁻⁴ moles/L:



So once the solution is saturated (reaches equilibrium):

$$[\text{Ag}^+] = 2.62 \times 10^{-4} \text{ moles/L}$$

$$[\text{CrO}_4^{2-}] = 1.31 \times 10^{-4} \text{ moles/L}$$

The Ksp expression is:

$$K_{sp} = [Ag^+]^2 [CrO_4^{2-}]$$

$$[Ag^+] = 2.62 \times 10^{-4} \text{ moles/L}$$

$$[CrO_4^{2-}] = 1.31 \times 10^{-4} \text{ moles/L}$$

Plugging in the concentrations directly we get:

$$K_{sp} = (2.62 \times 10^{-4})^2 (1.31 \times 10^{-4})$$

$$K_{sp} = \underline{8.99 \times 10^{-12}}$$

Our answer has 3 SD's because that was the lowest # of SD's given in the data. We did not use a Ksp table.

Now, here's a question for you to try:

- At a certain temperature, the *solubility* of SrCO₃ is 7.5 x 10⁻⁵ M. Calculate the *Ksp* for SrCO₃.

Answer _____

Check your answer on Page 4 of Tutorial 10 - Solutions

The next type of problem involves calculating the value of Ksp given the solubility in grams per Litre.

For this, you simply change grams/L to moles/L using the following:

$$\frac{\text{grams}}{\text{Litre}} \quad \times \quad \frac{1 \text{ mole}}{MM \text{ grams}} \quad = \quad \frac{\text{moles}}{\text{Litre}}$$

where "MM" stands for the Molar Mass.

Then write out the Net-Ionic equation for the equilibrium, and the Ksp expression.

Determine the concentrations of the ions and plug them into the K_{sp} expression to solve for K_{sp} .

Here's an example:

6.60 grams of MnF_2 will dissolve in one Litre of solution at $25^\circ C$. Calculate the value of K_{sp} for MnF_2 at $25^\circ C$.

Step 1- Change solubility in grams/L to moles/L using Molar Mass:

The molar mass of $MnF_2 = 54.9 + 2(19.0) = 92.9$ grams/mole

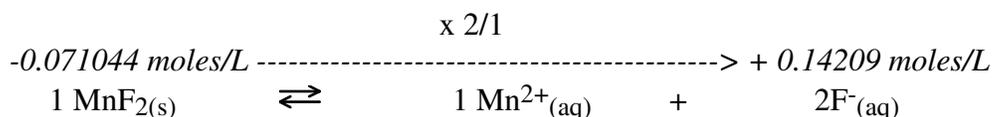
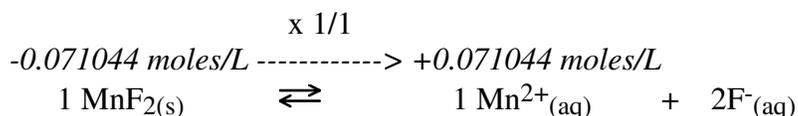
$$\frac{6.60 \text{ grams}}{\text{Litre}} \times \frac{1 \text{ mole}}{92.9 \text{ grams}} = \frac{0.071044 \text{ moles}}{\text{Litre}}$$

So, from step 1, the **molar solubility** of MnF_2 is 0.071044 moles/Litre.

Step 2- Write out the Net-Ionic Equation:



Step 3 - Write the molar solubility above the $MnF_{2(s)}$ and determine the $[Mn^{2+}]$ and $[F^-]$ using mole ratios:



It is important that you understand here that **$[F^-]$ is 0.14209, not the $[2F^-]$!**

($[F^-]$ happens to be **2 times** the solubility of MnF_2 just because of the mole ratio in the balanced equation.)

So $[Mn^{2+}] = 0.071044 \text{ M}$ and $[F^-] = 0.14209 \text{ M}$

Step 4 - Write out the K_{sp} expression:

$$K_{sp} = [Mn^{2+}] [F^-]^2 \quad (\text{not } [2F^-]^2)$$

Step 5 - "Plug-in" the values for $[Mn^{2+}]$ and $[F^-]$ and solve for the value of K_{sp} .

$$[Mn^{2+}] = 0.071044 \text{ M} \quad \text{and} \quad [F^-] = 0.14209 \text{ M}$$

$$K_{sp} = [Mn^{2+}] \times [F^-]^2$$

$$= (0.071044) (0.14209)^2$$

$$K_{sp} = \underline{1.43 \times 10^{-3}}$$

The lowest # of SD's in the data was 3 SD's. As we were working out the problem, we kept more SD's than this. Then, in the very last step, we rounded the answer to 3 SD's.

In summary, we have covered the following:

1. What is meant by K_{sp} .
2. How to write a " K_{sp} expression" from a *net ionic equation*.
3. How to calculate the *solubility* of an ionic substance in moles/L or in grams/L, given K_{sp} .
4. How to calculate the *value* of K_{sp} , given the *solubility* of an ionic substance.

Self-Test on Tutorial 10

Check your answers starting on page 5 of Tutorial 10 - Solutions

1. The K_{sp} is just a _____ for an ionic substance dissolving in water.
2. Give the *Net-Ionic Equation* and the *K_{sp} expression* for each of the following dissolving in water. (*The first one is done as an example.*)

Substance	Net-Ionic Equation	K_{sp} Expression
$Ag_2SO_4(s)$	$Ag_2SO_4(s) \rightleftharpoons 2Ag^+(aq) + SO_4^{2-}(aq)$	$K_{sp} = [Ag^+]^2 [SO_4^{2-}]$
$CaCO_3(s)$		
$Ca_3(PO_4)_2(s)$		
$AgClO_3(s)$		

3. a) Calculate the *molar solubility* (solubility in moles/Litre) of $\text{Fe}(\text{OH})_2$ in water.

Answer _____

- b) What is the $[\text{OH}^-]$ in a saturated solution of $\text{Fe}(\text{OH})_2$?

Answer _____

4. Calculate the *solubility* of BaCO_3 in grams per Litre.

Answer _____

5. The solubility of PbI_2 at a certain temperature is 0.70 grams per Litre.

a) Calculate the solubility in moles/Litre

Answer _____

b) Calculate the *value of K_{sp}* for PbI_2 at this temperature

Answer _____

6. It is found that 0.043 grams of MgCO_3 is all that can dissolve in 100.0 mL of solution at a certain temperature. From this information, calculate the *K_{sp}* for MgCO_3 at this temperature.

Answer _____

7. Two separate experiments were done with combinations of Cu^{2+} and IO_3^- ions. Use the information given to fill in the missing value.

The Net-Ionic Equation for equilibrium is: $\text{Cu}(\text{IO}_3)_2(\text{s}) \rightleftharpoons \text{Cu}^{2+}(\text{aq}) + 2\text{IO}_3^-(\text{aq})$

Experiment #	$[\text{Cu}^{2+}]$	$[\text{IO}_3^-]$
1	0.00327 M	0.00654 M
2	0.00240 M	?

Check your answers starting on page 5 of Tutorial 10 - Solutions

This is the End of Tutorial 10