Chemistry 12
Tutorial 10
Ksp Calculations

Welcome back to the world of calculations.
In Tutorial 10 you will be shown:

1. What is meant by ***Ksp*** .
2. How to write a "***Ksp expression***" from a *net ionic equation*.
3. How to calculate the ***solubility*** of an ionic substance in moles/L or in grams/L, given the value of Ksp .
4. How to calculate the ***value*** of Ksp , given the *solubility* of an ionic substance.

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What is Ksp ?

Ksp is really nothing new. It's simply the equilibrium constant (K_{eq}) for ***an ionic substance dissolving in water***.

For example, for the substance, $CaCO_{3(s)}$, when it dissolves in water, forms the ions Ca^{2+} and CO_3^{2-}.

The equilibrium ***net-ionic equation*** for this is: $CaCO_{3(s)} \rightleftarrows Ca^{2+}_{(aq)} + CO_3^{2-}{}_{(aq)}$.

The equilibrium constant (K_{eq}) expression for this reaction is:

$K_{eq} = [Ca^{2+}] [CO_3^{2-}]$ (*Remember that the [$CaCO_3$] is **not** included in the expression because it is a **solid**.*)

Remember that the ***solubility*** was defined as *the equilibrium concentration of a substance in water.*

But when ionic compounds dissolve in water, you always get *at least 2 ions* formed. (In this case the Ca^{2+} and the CO_3^{2-}.)

Multiplying the concentrations of the two ions (multiplication gives a "*product*"), as you can see, gives the K_{eq} expression:

$$K_{eq} = [Ca^{2+}] \ [CO_3^{2-}]$$

K_{eq} can be thought of as the "product of the solubility's of the two ions". So for ionic compounds dissolving in water, the K_{eq} is given a special name:
It is called "***solubility product constant***", or ***Ksp*** .

Writing the Ksp expression from the Net-Ionic Equation.

There's really nothing new to this. Just remember to ***leave out the solid***. (*Include the aqueous ions only*) and to ***change coefficients in the balanced equation to exponents*** in the Ksp expression.

Here are a couple of examples:

The net-ionic equation for lead (II) chloride dissolving is:

$PbCl_{2(s)} \rightleftarrows Pb^{2+}{}_{(aq)} + 2Cl^{-}{}_{(aq)}$

The Ksp expression for this is:

$Ksp = [Pb^{2+}] [Cl^{-}]^2$ (*Notice that the "2" in front of the Cl^- in the equation becomes an exponent in the Ksp expression. It does NOT go in front of the Cl^- in the Ksp expression!*)

The net-ionic equation for silver sulphate dissolving is:

$Ag_2SO_{4(s)} \rightleftarrows 2Ag^{+}{}_{(aq)} + SO_4{}^{2-}{}_{(aq)}$

The Ksp expression would be:

$Ksp = [Ag^+]^2 [SO_4{}^{2-}]$

I don't think you should have any more trouble with these as long as you follow the simple rules:

1. ***leave out the solid***. (*Include the aqueous ions only*)

2. ***change coefficients in the balanced equation to exponents in the Ksp expression.***

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Calculating Solubility given Ksp .

The first type of calculation we will look at is how to calculate the solubility of a substance in moles per litre (M), given the value of Ksp .

NOTE: We only consider the Ksp and the solubility of substances which have "***Low Solubility***" on the Solubility Table. These are also called "***Slightly Soluble Salts***"

For these calculations, we can define ***molar solubility*** as *the moles of the substance which will dissolve in one litre of solution to form a saturated solution.*

On the next page, you will be shown how to calculate the ***molar solubility*** of AgCl from the Ksp.

First we obtain a sheet entitled "Solubility Product Constants at 25 ºC" This has Ksp's for many of the "Low Solubility Compounds" listed. Always have this table with you on a test!

So, we find that the Ksp for AgCl = 1.8×10^{-10}

The net-ionic equation for the substance, AgCl dissolving in water is:

$$\overset{-s}{AgCl_{(s)}} \rightleftarrows Ag^{+}_{(aq)} + Cl^{-}_{(aq)}$$

The number of *moles* of AgCl which dissolve in *one litre* can be defined as "*s*". (*This is the molar solubility*.)

Notice, that this is written as "-*s*" above AgCl in the equation. This is because when you dissolve AgCl in water, the amount of solid will go *down* by "s" because *that much* *dissolves*. The "-s" is the same thing as "[C]" or "change in concentration" in an ICE table.

Now by mole ratios, you can see, that in this case [Ag^+] will *increase* by "*s*" as the AgCl dissolves. Also, the [Cl^-] will also *increase* by "*s*". (*The coefficients of Ag^+ and Cl^- are both "1".*)

$$\overset{-s}{AgCl_{(s)}} \xrightarrow{1/1} \overset{+s}{Ag^{+}_{(aq)}} + Cl^{-}_{(aq)}$$

and

$$\overset{-s}{AgCl_{(s)}} \rightleftarrows Ag^{+}_{(aq)} + \overset{+s}{Cl^{-}_{(aq)}} \quad (-s \xrightarrow{1/1} +s)$$

so we have:

$$\overset{-s}{AgCl_{(s)}} \rightleftarrows \overset{+s}{Ag^{+}_{(aq)}} + \overset{+s}{Cl^{-}_{(aq)}}$$

where "s" is the moles/L of AgCl which dissolve.

The Ksp expression for this process is:

Ksp = [Ag^+] [Cl^-]

What we do now is "plug in" the values of [Ag^+] and [Cl^-] in terms of "s". These are the expressions written on top of the net-ionic equation:

[Ag^+] = s and

[Cl^-] = s (*we don't need to write the "+" signs)*

Since: $Ksp = [Ag^+] [Cl^-]$

$Ksp = (s)(s)$

or $Ksp = s^2$

Now we can solve for s. Remember we found from the Ksp table that the value for $Ksp = 1.8 \times 10^{-10}$

So $s^2 = 1.8 \times 10^{-10}$ or s $\sqrt{1.8 \times 10^{-10}}$ =
= 1.34164×10^{-5} M

Since "s" was defined as the ***molar solubility*** of AgCl:

$$\overset{-s}{AgCl_{(s)}} \rightleftarrows \overset{+s}{Ag^+_{(aq)}} + \overset{+s}{Cl^-_{(aq)}}$$

where "s" is the moles/L of AgCl which dissolve.

We can say, ***the molar solubility of AgCl*** is 1.34164×10^{-5} moles/L or
1.3×10^{-5} M (*2 sig. digs.*) (*The Ksp given on the table has 2 SD's*)

<u>NOTE</u>: It looks awful tempting at this point to just say that the solubility is the square root of the Ksp . It is in this case, but not in all cases.

<u>Make sure when you do these problems that you always write</u>:

1. The net-ionic equilibrium equation (with the "s"s on top according to the coefficients),

2. The Ksp expression, and

3. The concentrations plugged into the Ksp expression to solve for "s".

DON'T TAKE SHORT CUTS! They could lead to **trouble** in the next type of problem!

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The next problem, you should try on your own, keeping the last three points in mind. Don't worry too much if you get it wrong the first time. Just make sure you understand the solution given on page 1 of Tutorial 10 - Solutions.

1. Calculate the solubility of SrF_2 in moles/Litre in water.

Answer ______________________

<u>Check your answer on page 1 of Tutorial 10 – Solutions</u>

Now, let's say you're given the Ksp table and asked to find the solubility of a substance in grams/L.

What you would do is determine the solubility in moles/L (or M) first, then convert to grams/L using the following:

$$\frac{\text{moles}}{\text{Litre}} \quad \text{x} \quad \frac{MM \text{ grams}}{1 \text{ mole}} = \frac{\text{grams}}{\text{Litre}}$$

Where "*MM*" stands for the Molar Mass.

Try this example:

2. Calculate the solubility of Ag_2CO_3 in grams/Litre.

Answer ______________________________

Check your answer on page 2 of Tutorial 10 - Solutions

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Calculating Ksp , Given Solubility

The first example we will do is to calculate the Ksp of a substance given it's molar solubility.

In this type of problem we don't use "s"s. The molar solubility is known, so we find the concentration of each ion using mole ratios (record them on top of the equation). Next we write out the expression for Ksp , then "plug in" the concentrations to obtain the value for Ksp.

Let's do an example:

The solubility of Ag_2CrO_4 in water is 1.31 x 10^{-4} moles/L. Calculate the value of Ksp .

The first thing we do is write out the net-ionic equation for a saturated solution of Ag_2CrO_4:

$$Ag_2CrO_{4\,(s)} \rightleftarrows 2Ag^+_{(aq)} + CrO_4^{2-}{}_{(aq)}$$

The solubility given is 1.31 x 10^{-4} moles/L, so we write that right on top of the $Ag_2CrO_{4(s)}$
(*I usually write it as a negative (-) because it is going down (the solid is dissolving)*)

$$\underset{Ag_2CrO_{4\,(s)}}{\textit{-1.31 x }10^{-4}\textit{ moles/L}} \rightleftarrows 2Ag^+_{(aq)} + CrO_4^{2-}{}_{(aq)}$$

Using mole ratios, the [Ag^+] will go up by (*2 x* 1.31 x 10^{-4} moles/L) = 2.62 x 10^{-4} moles/L.

$$\textit{-1.31 x }10^{-4}\textit{ moles/L} \xrightarrow{\text{x 2/1}} \textit{+ 2.62 x }10^{-4}\textit{ moles/L}$$
$$Ag_2CrO_{4\,(s)} \rightleftarrows 2Ag^+_{(aq)} + CrO_4^{2-}{}_{(aq)}$$

[CrO_4^{2-}] will go up by 1.31 x 10^{-4} moles/L:

$$\textit{-1.31 x }10^{-4}\textit{ moles/L} \xrightarrow{\text{x 1/1}} \textit{+ 1.31 x }10^{-4}\textit{ M.}$$
$$Ag_2CrO_{4\,(s)} \rightleftarrows 2Ag^+_{(aq)} + CrO_4^{2-}{}_{(aq)}$$

So once the solution is saturated (reaches equilibrium):

[Ag^+] = 2.62 x 10^{-4} moles/L

[CrO_4^{2-}] = 1.31 x 10^{-4} moles/L

The Ksp expression is:

$Ksp = [Ag^+]^2 [CrO_4^{2-}]$

$[Ag^+] = 2.62 \times 10^{-4}$ *moles/L*
$[CrO_4^{2-}] = 1.31 \times 10^{-4}$ *moles/L*

Plugging in the concentrations directly we get:

$Ksp = (2.62 \times 10^{-4})^2 (1.31 \times 10^{-4})$

Ksp = $\underline{8.99 \times 10^{-12}}$

Our answer has 3 SD's because that was the lowest # of SD's given in the data. We did not use a Ksp table.

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Now, here's a question for you to try:

3. At a certain temperature, the ***solubility*** of $SrCO_3$ is 7.5×10^{-5} M. Calculate the ***Ksp*** for $SrCO_3$.

Answer ___________________

<u>Check your answer on Page 4 of Tutorial 10 - Solutions</u>

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The next type of problem involves calculating the value of Ksp given the solubility in grams per Litre.

For this, you simply change grams/L to moles/L using the following:

$$\frac{\text{grams}}{\text{Litre}} \times \frac{1 \text{ mole}}{MM \text{ grams}} = \frac{\text{moles}}{\text{Litre}}$$

where "MM" stands for the Molar Mass.

Then write out the Net-Ionic equation for the equilibrium, and the Ksp expression.

Determine the concentrations of the ions and plug them into the Ksp expression to solve for Ksp .

Here's an example:

6.60 grams of MnF_2 will dissolve in one Litre of solution at 25°C. Calculate the value of Ksp for MnF_2 at 25°C.

Step 1- Change solubility in grams/L to moles/L using Molar Mass:

The molar mass of MnF_2 = 54.9 + 2(19.0) = 92.9 grams/mole

$$\frac{6.60\ \cancel{\text{grams}}}{\text{Litre}} \times \frac{1\ \text{mole}}{92.9\ \cancel{\text{grams}}} = \frac{0.071044\ \text{moles}}{\text{Litre}}$$

So, from step 1, the ***molar solubility*** of MnF_2 is 0.071044 moles/Litre.

Step 2- Write out the Net-Ionic Equation:

$$MnF_{2(s)} \rightleftarrows Mn^{2+}_{(aq)} + 2F^-_{(aq)}$$

Step 3 - Write the molar solubility above the $MnF_{2(s)}$ and determine the $[Mn^{2+}]$ and $[F^-]$ using mole ratios:

$$\begin{array}{ccccc} & \times 1/1 & & & \\ -0.071044\ \text{moles/L} & \longrightarrow & +0.071044\ \text{moles/L} & & \\ 1\ MnF_{2(s)} & \rightleftarrows & 1\ Mn^{2+}_{(aq)} & + & 2F^-_{(aq)} \end{array}$$

$$\begin{array}{ccccc} & & \times 2/1 & & \\ -0.071044\ \text{moles/L} & & \longrightarrow & & +\ 0.14209\ \text{moles/L} \\ 1\ MnF_{2(s)} & \rightleftarrows & 1\ Mn^{2+}_{(aq)} & + & 2F^-_{(aq)} \end{array}$$

It is important that you understand here that **$[F^-]$ is 0.14209**, ***not*** the ~~$[2F^-]$~~ !

(*$[F^-]$ happens to be* **2 times** *the solubility of MnF_2 just because of the* ***mole ratio*** *in the balanced equation.*)

So $[Mn^{2+}]$ = 0.071044 M and $[F^-]$ = 0.14209 M

Step 4 - Write out the Ksp expression:

Ksp = $[Mn^{2+}]\ [F^-]^2$ (***not*** *$[2F^-]^2$*)

Step 5 - "Plug-in" the values for $[Mn^{2+}]$ and $[F^-]$ and solve for the value of Ksp .

$[Mn^{2+}] = 0.071044$ M and $[F^-] = 0.14209$ M

$$Ksp = [Mn^{2+}] \times [F^-]^2$$
$$= (0.071044)\ (0.14209)^2$$

$$Ksp = \underline{\boldsymbol{1.43 \times 10^{-3}}}$$

The lowest # of SD's in the data was 3 SD's. As we were working out the problem, we kept more SD's than this. Then, in the very last step, we rounded the answer to 3 SD's.

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In summary, we have covered the following:

1. What is meant by ***Ksp*** .
2. How to write a "***Ksp expression***" from a *net ionic equation*.
3. How to calculate the ***solubility*** of an ionic substance in moles/L or in grams/L, given Ksp .
4. How to calculate the ***value*** of Ksp , given the *solubility* of an ionic substance.

Self-Test on Tutorial 10

Check your answers starting on page 5 of Tutorial 10 - Solutions

1. The Ksp is just a __________ for an ionic substance dissolving in water.
2. Give the ***Net-Ionic Equation*** and the ***Ksp expression*** for each of the following dissolving in water. (*The first one is done as an example*.)

Substance	Net-Ionic Equation	Ksp Expression
$Ag_2SO_{4(s)}$	$Ag_2SO_{4(s)} \rightleftarrows 2Ag^+_{(aq)} + SO_4^{2-}{}_{(aq)}$	$Ksp = [Ag^+]^2\ [SO_4^{2-}]$
$CaCO_{3(s)}$		
$Ca_3(PO_4)_{2(s)}$		
$AgClO_{3(s)}$		

3. a) Calculate the ***molar solubility*** (solubility in moles/Litre) of $Fe(OH)_2$ in water.

Answer ______________________

b) What is the $[OH^-]$ in a saturated solution of $Fe(OH)_2$?

Answer ______________________

4. Calculate the ***solubility*** of $BaCO_3$ in grams per Litre.

Answer ______________________

5. The solubility of PbI_2 at a certain temperature is 0.70 grams per Litre.

   a) Calculate the solubility in moles/Litre

   Answer ______________________

   b) Calculate the ***value of Ksp*** for PbI_2 at this temperature

   Answer ______________________

6. It is found that 0.043 grams of $MgCO_3$ is all that can dissolve in 100.0 mL of solution at a certain temperature. From this information, calculate the ***Ksp*** for $MgCO_3$ at this temperature.

   Answer ______________________

7. Two separate experiments were done with combinations of Cu^{2+} and IO_3^- ions. Use the information given to fill in the missing value.

   The Net-Ionic Equation for equilibrium is: $Cu(IO_3)_{2(s)} \rightleftarrows Cu^{2+}_{(aq)} + 2IO_3^-{}_{(aq)}$

Experiment #	$[Cu^{2+}]$	$[IO_3^-]$
1	0.00327 M	0.00654 M
2	0.00240 M	?

Check your answers starting on page 5 of Tutorial 10 - Solutions

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This is the End of Tutorial 10